System priors for econometric time series

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Disclaimer

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Aims and scope

- To provide more nuanced and more general introduction to system priors (devised by Andrle and Beneš within the DSGE context)
- To demonstrate the generality of principles and its wide scope of application
- To illustrate the use of system priors with a simple but practically relevant example
- ... to invite fellow researchers to jump on the bandwagon

What are system priors?

- Economically-meaningful priors about high-level model properties
  - impulse-response functions
  - variance error decompositions
  - frequency-domain properties
  - sacrifice ratios
  - ...anything that can be computed with the model
- Two layer approach that facilitates formulation of priors on both the parameter and model level
- Complement rather than substitute for traditional Bayesian setup

Why and when one should use system priors?

- In complex models individual parameters are difficult to interpret.
- Reasonable priors for individual parameters may lead in sum to highly erratic priors about the overall model behavior.
  - Even non-informative priors can be implicitly very informative in a highly undesirable way
  - Prior predictive analysis – which parameter priors “bite”?
- Policy makers only hold firm views about the economic behavior.
  - Communication channel between modelers and policy makers

First glance at system priors

- Traditional bayesian setup

$$p(\vartheta / Y;M) \propto L(Y / \vartheta;M) \times p_m(\vartheta)$$

- System priors setup

$$p(\vartheta / Y;M) \propto L(Y / \vartheta;M) \times [\, p_s(h(\vartheta);M) \times p_m(\vartheta)]$$

  - $p_m(\vartheta)$ – priors on individual parameters
  - $p_s(h(\vartheta);M)$ – system priors „add-on“
  - $[p_s(h(\vartheta);M) \times p_m(\vartheta)]$ – composite prior enabling to implement views on elements in both layers

How to understand system priors I

- **(Non-conjugate) dummy observation prior**
  - Instead of inserting dummy observations into the dataset, create a dummy/artificial likelihood (for the auxiliary model) that summarizes the information in the dummy observations
- $[\, p_s(h(\vartheta);M) \times p_m(\vartheta)] \equiv \mathit{likelihood} \times \mathit{prior\ on\ parameters}$
- Posterior inference is obtained by updating priors on individual parameters twice:
  - first with artificial likelihood of the auxiliary model (system priors)
  - second with real likelihood based on observed data

How to understand system priors II

- **Penalized likelihood problem**
  - Taking logs of the RHS...

$$p(\vartheta / Y;M) \propto L(Y/\vartheta;M) \times [\, p_s(h(\vartheta);M) \times p_m(\vartheta)]$$

  - ... one obtains

$$\log(L(Y/\vartheta;M)) + \log\left(p_m(\vartheta)\right) + \log(p_s(h(\vartheta);M)$$

- Finding the mode of the posterior distribution is a traditional maximum likelihood approach with additional penalties that "regularize" the problem

- Penalty terms are nothing new in econometrics
  - ridge regression
  - lasso
  - many others...

Related literature I

- A desire for a priori constraints on model properties in not new, however most of the existing attempts only have *ad hoc* nature
  - priors only solve specific a problem at hand (e.g. steady-state priors – Villani, 2005; priors on impulse responses – Dwyer, 1998, Kocięcki, 2012; long-run priors – Giannone et al., 2016; priors on frequencies – Planas et al., 2008)
  - priors only take specific form (usually gaussian priors)

- More general approaches
  - *Feature of interest priors*: Hollifield et al. (2003) – this approach is conceptually identical to system priors
  - *Priors on observables*: Jarociński and Marcet (2013)

Related literature II

Comparison of our approach with that of Jarociński and Marcet

- Both approaches can be used to solve similar problems, however they differ in concept (and flexibility & versatility).

- Both approaches have to solve the inverse problem:

- Jarociński and Marcet
  - Priors on high-level features -> Priors on observables -> Fredholm equation/fixed point solution -> implied priors on individual parameters -> bayesian update (likelihood) -> posterior distribution

- System priors
  - Priors on individual parameters -> bayesian update (artificial likelihood) -> bayesian update (likelihood) -> posterior distribution

Illustrative example

- Stationary AR(2) process with additional belief that most of its variance is generated by business-cycle frequencies
  - AR(2) is a very simple case, but the process can exhibit non-trivial dynamics
- We use the example only as an illustration, however it can be quite useful for empirical work
  - output gaps are frequently modelled as the AR(2) process: (see e.g. Watson, 1986, Clark, 1987, Kuttner, 1994, Planas et al., 2008, Jarociński and Lenza, 2016 and many others)
  - the same goes for inflation gaps (Clark and Doh, 2014)
  - ...or unemployment gaps (Chan et al., 2016)

Illustrative example

If the AR(2) is used to capture some "business-cycle-gap" variable, what are the options to consider?

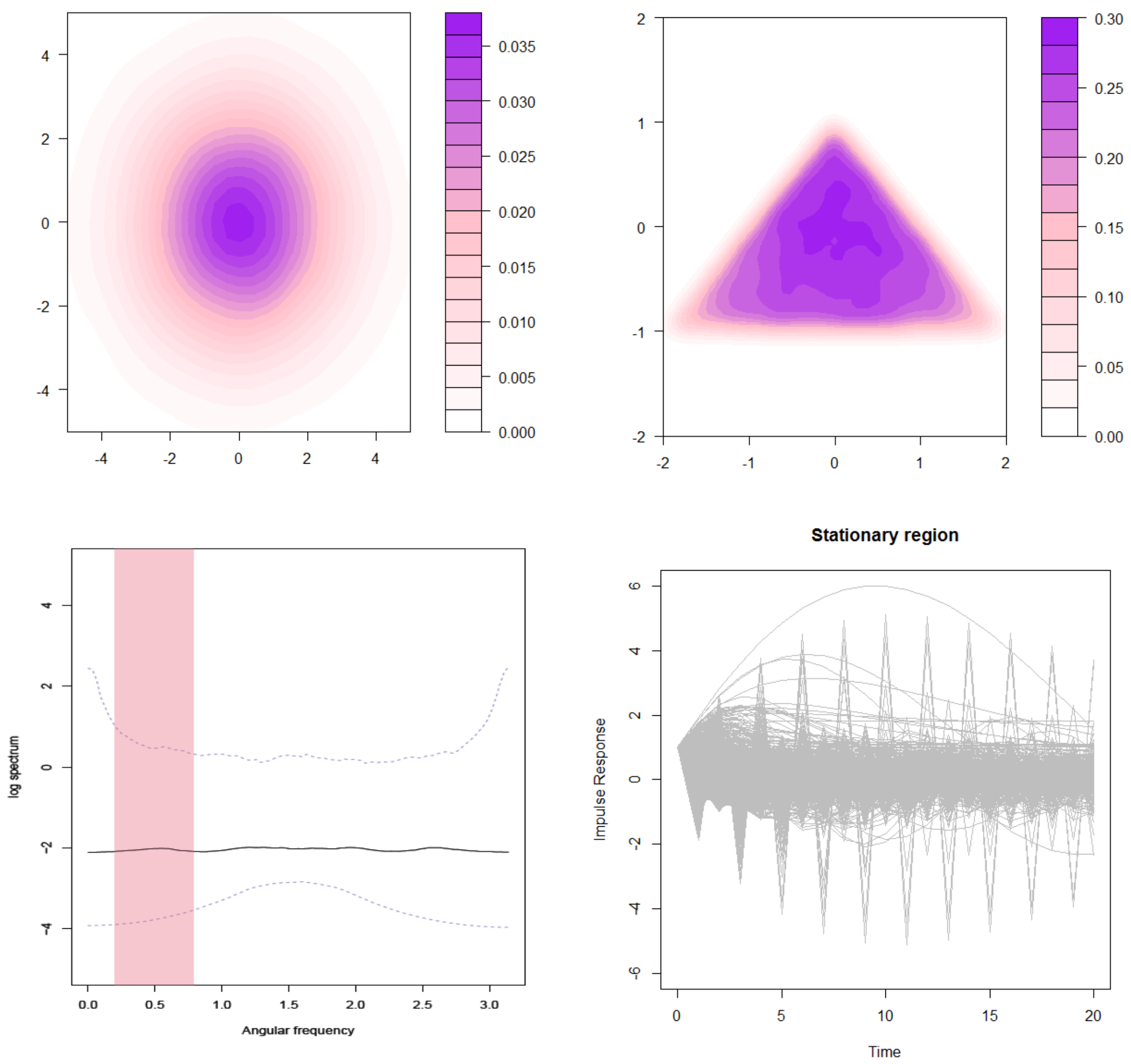
- basic Gaussian option: Chan and Grant(2017); earlier versions of Chan et al. (2015)
- model reparametrization: Planas et al. (2008)
- more refined Gaussian option ("gamekeeper's trick"): Chan et al. (2015); Grant and Chan (2017); Lenza and Jarociński (2016)
- System priors based on the business-to-total-variance ratio
  - at least 60% of variance comes from business cycle frequencies
  - the ratio follows some distribution [Be(15,5) is used in the paper]

$$ratio = \int_a^b S_y(w)\mathrm{d}w \Big/ \int S_y(w)\mathrm{d}w\,,$$

$S_y(w)$ – spectral density of the process

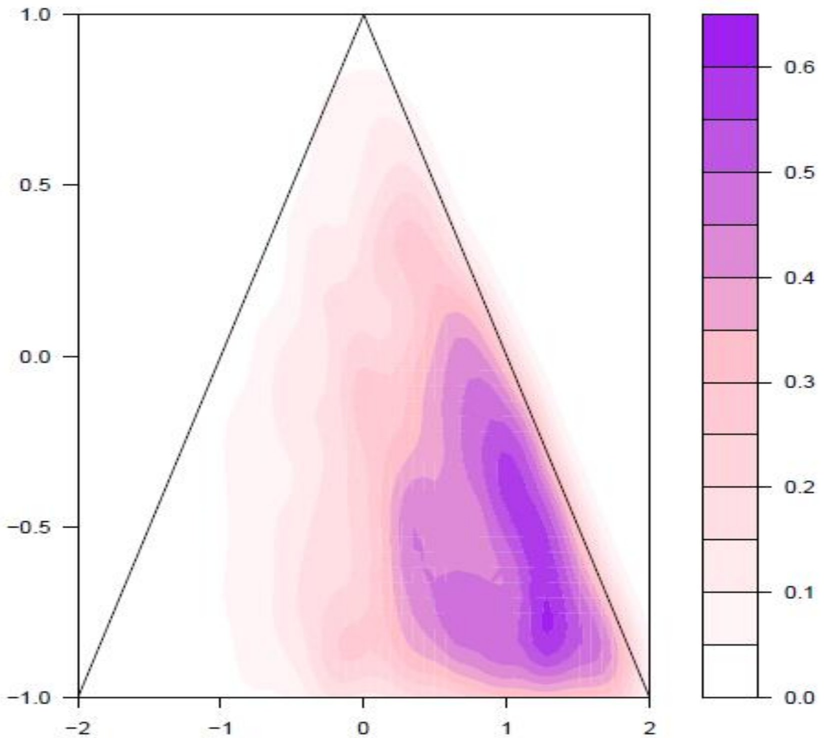
a,b – limits for business cycle frequencies

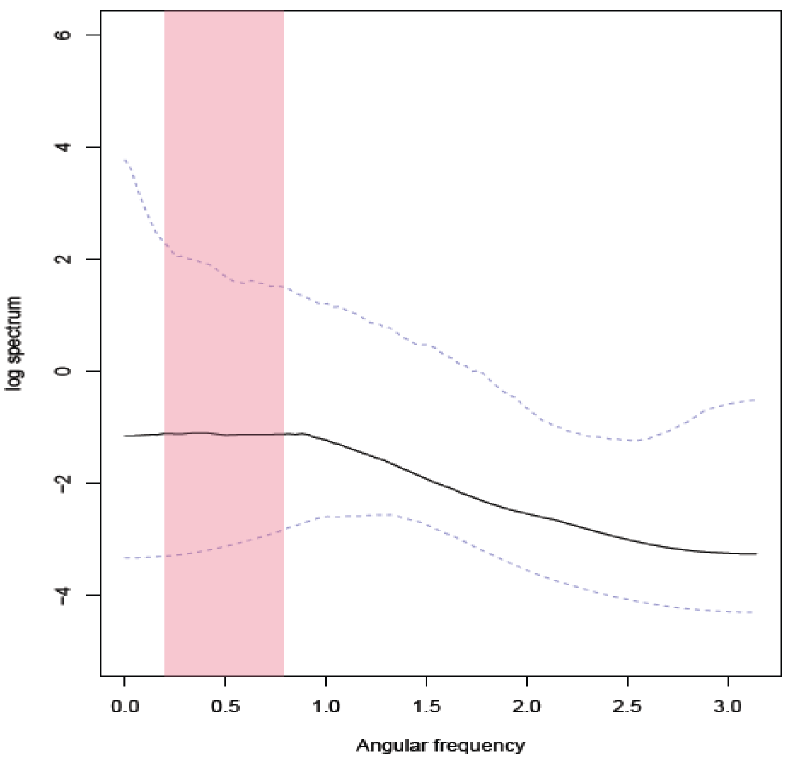
Basic Gaussian option



More refined Gaussian option I

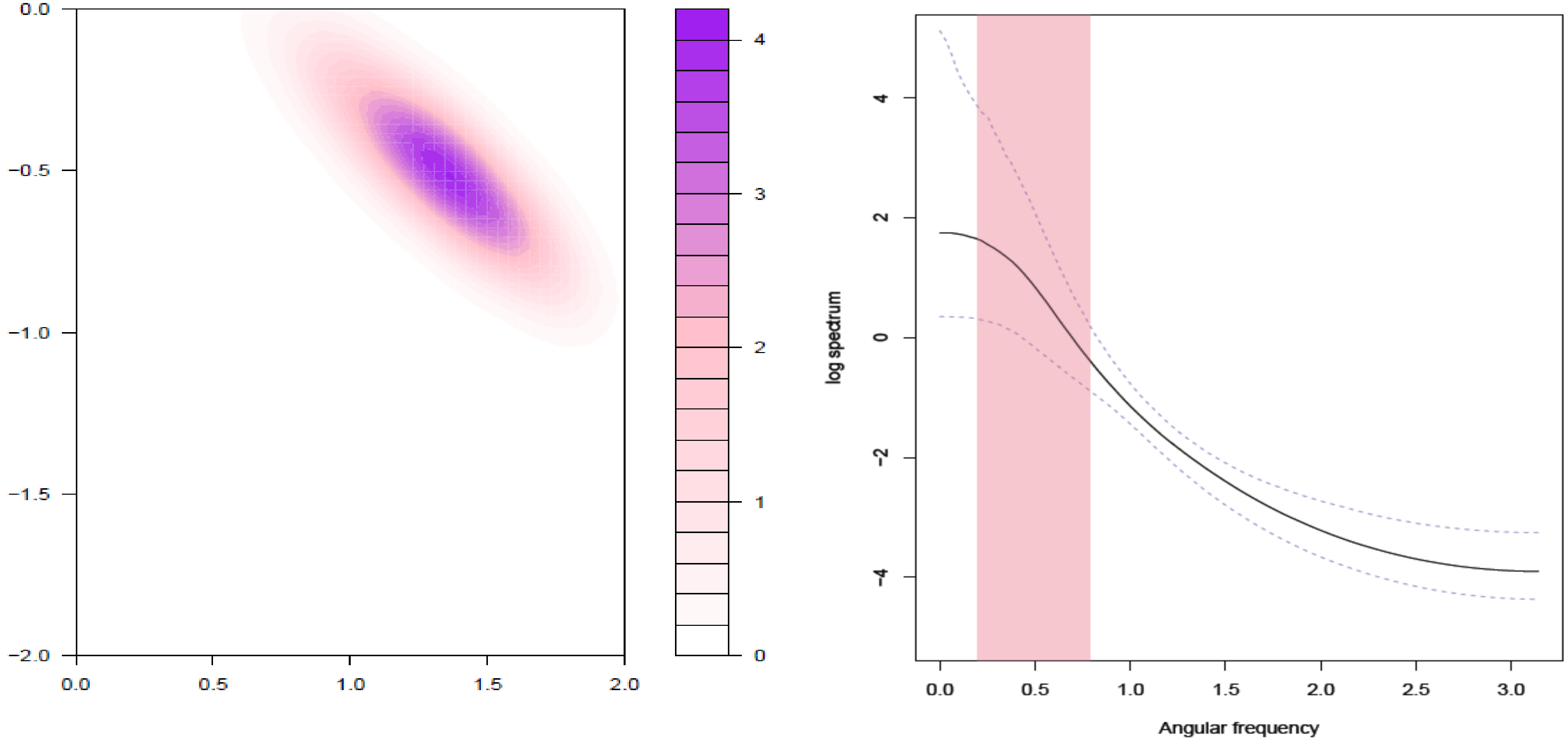
Grant and Chan (2017): $N\left(\begin{pmatrix} 1.3 \\ -0.7 \end{pmatrix}, I(2)\right)$



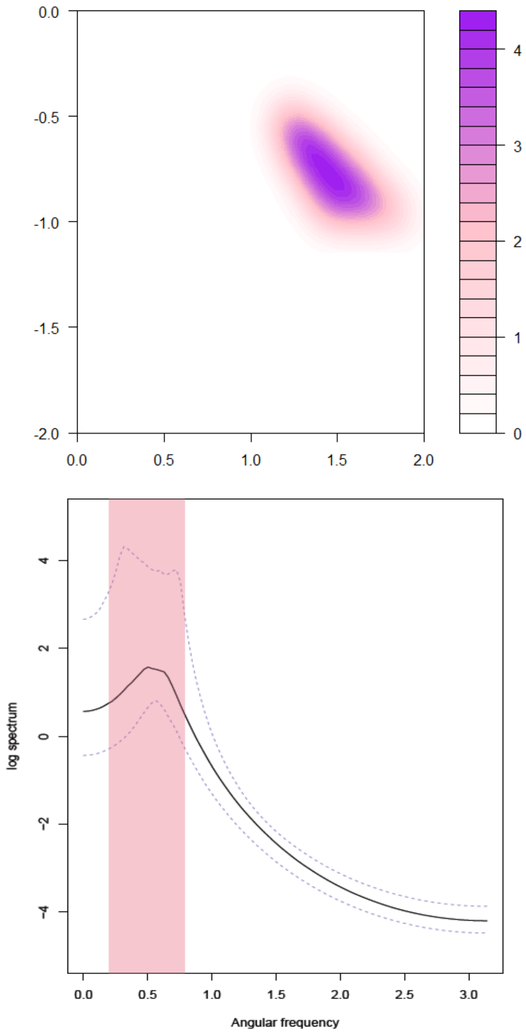


More refined Gaussian option II

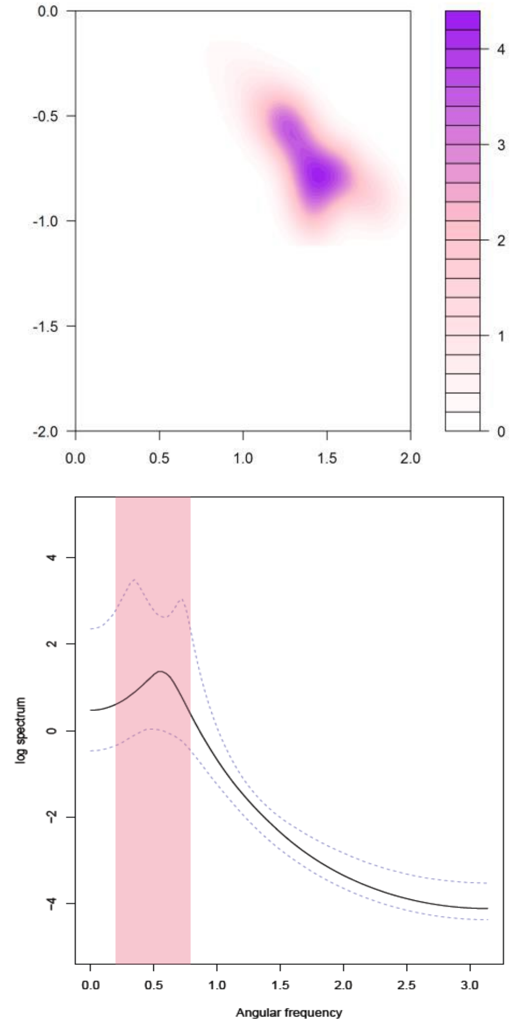
Lenza and Jarociński (2016): $N\left(\begin{pmatrix}1.352\\-0.508\end{pmatrix},\begin{bmatrix}0.0806 & -0.0578\\-0.0578 & 0.0464\end{bmatrix}\right)$



System priors



At least 60 %



Be(15,5)

Conclusions

- System priors represent a flexible way of incorporating economically meaningful information.
- They are very general and can be easily implemented within existing Bayesian toolkit.
- The paper places emphasis on the elements and mechanics of system priors' application.
- Implementation of system priors was illustrated using second-order autoregressive process and constraints on stationarity and frequency-domain properties.
- Next stop: system priors for VARs

Thank you for your attention

- Q & A section
  - Questions and comments are more than welcome!
- Discussion

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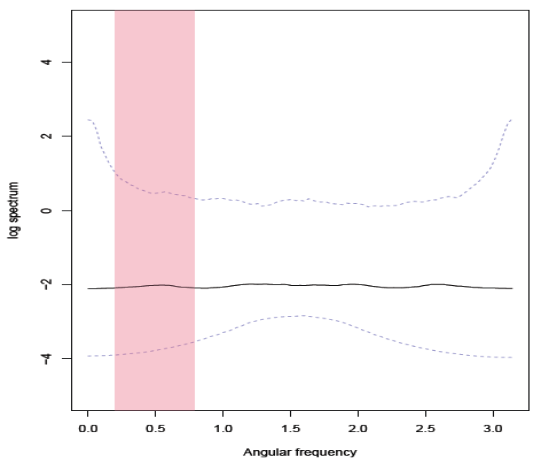
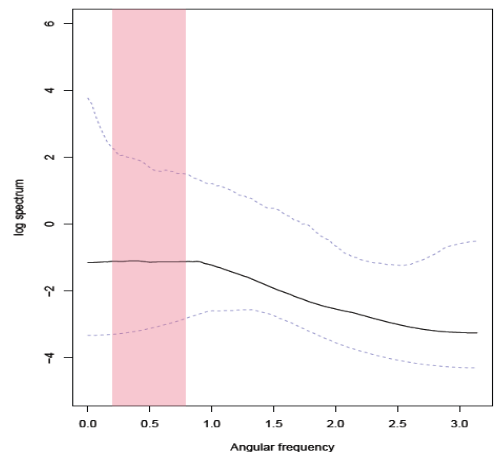
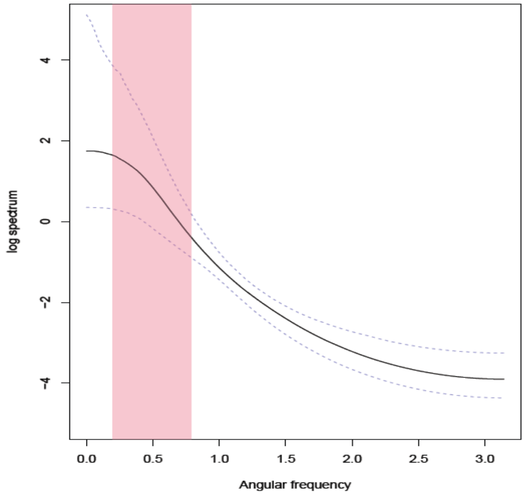
Back-up slides 1



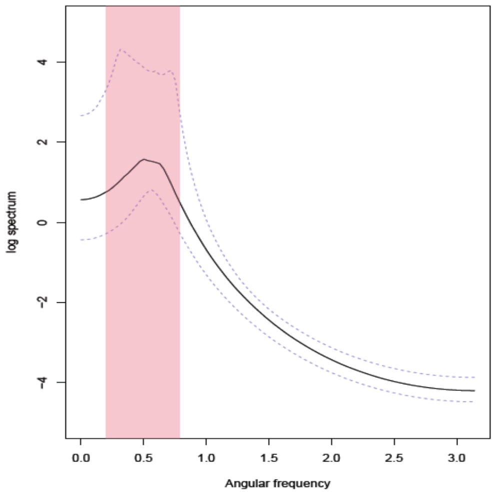
Stationary AR(2)



Chan et al. (2015)

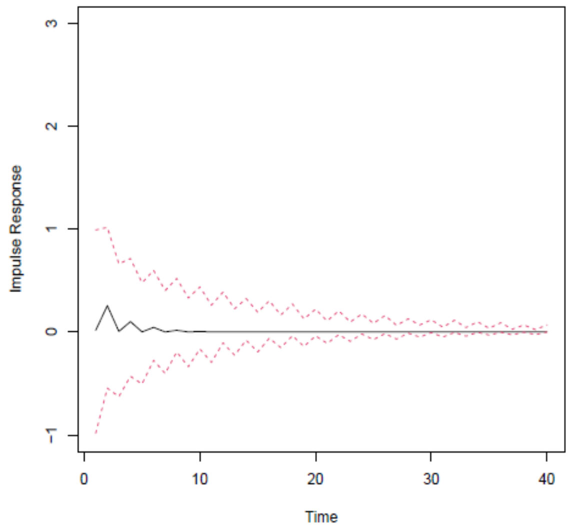


Lenza and Jarociński (2016)

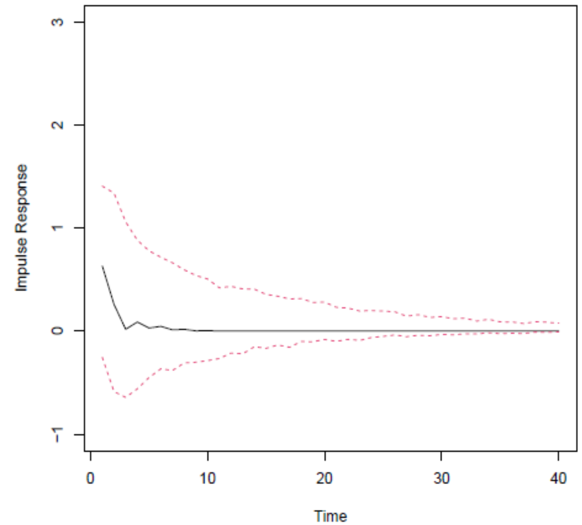


System priors, at least 60%

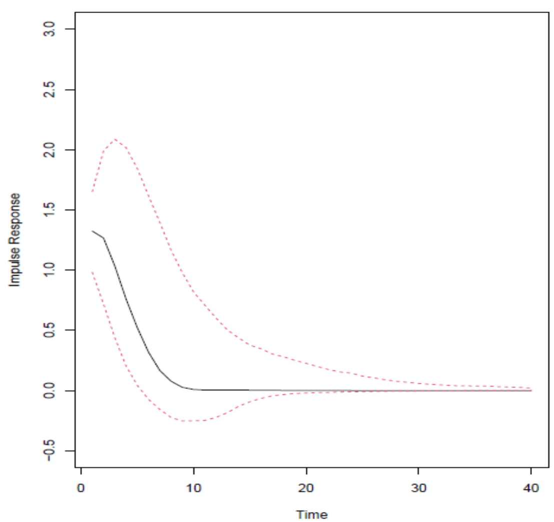
Back-up slides 2: Impulse response functions



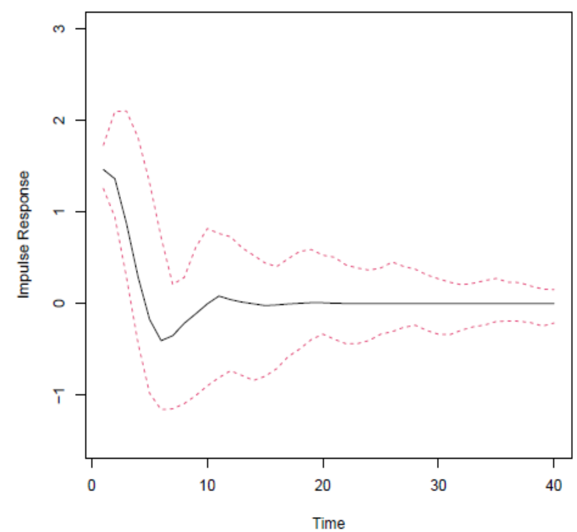
Stationary AR(2)



Chan et al. (2015)



Lenza and Jarociński (2016)



System priors, at least 60%

Back-up slides 2: System priors – alternatives